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# Tiering and Scaffolding:

## Two Strategies for Providing Access to Important Mathematics

**J**oe is a wonderful artist. He can visualize pictures in his mind and then draw them with great detail. He reads at grade level but seems frustrated by mathematics and is still working to develop fluency with whole-number computation. Chue loves mathematics. Numbers make sense to him, and the language barrier does not seem quite so overwhelming when mathematics class begins. Maria always follows directions and does her work quickly and quietly. She reads above grade level and easily understands patterns and relationships with numbers when new concepts are introduced in mathematics class.

**By Lori Williams**

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The diversity of students in today's mathematics classroom is impossible to ignore. Providing for equity in the classroom—an appropriate level of challenge with the appropriate supports (NCTM 2000)—for all the Joes, Chues, and Marias can become a daunting task when we consider the wide variety of students' readiness levels for important mathematics concepts and skills taught each year. *Tiering* and *scaffolding* are two differentiation strategies that teachers use to support equitable access to a high-quality mathematics curriculum. The examples that follow demonstrate the thinking process teachers follow to plan tiered learning activities that focus on the important mathematical understandings of the lessons. The examples also describe possible *scaffolds*, or temporary supports for learning, that can be used to allow students to work at an appropriate level of challenge with the confidence that they know what they are doing and how they are supposed to do it (ASCD 2002).

## Tiering Tasks

Tomlinson and Eidson (2003) define *tiering* as a process of adjusting the degree of difficulty of a question, task, or product to match a student's current readiness level (p. 190). They recommend that teachers follow these steps to create tiered tasks:

1. Determine what students should know, understand, and be able to do as a result of the task.
2. Consider the students' readiness range relevant to those goals.
3. Develop or select an activity that is interesting, requires high-level thought, and causes students to work with the specified knowledge, understanding, and skill.
4. Determine the complexity level of that starting-point task compared with the range of student readiness.
5. Develop multiple versions of the task at different levels of difficulty, ensuring that all versions focus on essential knowledge, understanding, and skill.
6. Assign students to the various versions of the task at levels likely to provide attainable challenge.

## Tiering a Third-Grade Fractions Lesson

Sally applied the steps for tiering while planning a third-grade mathematics lesson on fractions. First, she identified what she would like the students to understand and be able to do. Part of the third-grade curriculum requires students to understand and represent commonly used fractions when they are used in a context and to recognize fractions as equal parts of a whole unit.

Next, Sally thought about third graders and their understanding of fractions. Young children with a developing number sense of fractions must understand that when dividing whole units into fractional parts, the entire unit must be used up, the parts have to be equal, and some part of the whole must be given to each sharer. Many students have an informal knowledge of "halving" and "repeated halving" on which they can build to divide whole units among two, four, and eight sharers before moving to dividing among three or five sharers (Empson 2002). As Sally watched her students work with the first sharing activities, she found that some students divided units into parts without making sure the parts were equal, and a few students cut out the correct number of pieces from a brownie and threw the extra away. Some students were only comfortable

**Figure 1**

Sally's students cut paper "brownies" into fractional pieces.



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dividing units into two parts, some were ready to move to dividing units into four or eight parts, and some appeared to be ready for dividing units into any number of parts.

Third, Sally identified an interesting activity to cause the students to use higher-level thinking skills as they constructed their knowledge of fractions. She found such an activity, "More Sharing Problems," a unit of fraction activities in *Fair Shares* (Tierney and Berle-Carman 1998). This activity interested the students because it reflected a situation that they encounter outside school when they have a limited amount of a resource (e.g., brownies) and need to share the resource among their friends. The activity asked students to cut sets of paper brownies into fractional pieces and share them equally (see **fig. 1**). They were provided a recording sheet on which to document their work as pictorial representations of the shares and their answers as symbolic representations of the shares. The manual provided Sally with a list of possible situations her students could investigate:

- How can 2 people share 3 brownies?
- How can 2 people share 5 brownies?
- How can 3 people share 4 brownies?
- How can 3 people share 5 brownies?
- How can 4 people share 2 brownies?
- How can 4 people share 3 brownies?

- How can 3 people share 2 brownies?
- How can 6 people share 4 brownies?
- How can 5 people share 4 brownies?

When Sally compared the level of complexity of this task with the range of readiness of her third graders, she asked herself the following questions:

- Who would find this task too easy? Why?
- Who would find this task too difficult? Why?

Students like Maria and Chue, who could easily use repeated halving, would find a number of the situations simple to think through, whereas students like Joe, who was still developing an understanding of halves as two equal parts, might struggle with most of the questions.

To create multiple versions of the task, Sally grouped the questions into sets on the basis of their complexity. She created task cards (see **fig. 2**) to meet the needs of three levels of learners. The first problems on the first card represented situations that helped students who still needed to build on their informal understanding of halving to create

equal parts. The successive problems gave them an opportunity to use repeated halving and then move to dividing the whole units into thirds. The second card contained problems for those children who were already comfortable with the repeated halving strategy. Their problems helped them develop strategies that did not involve halving. Finally, the situations on the third card asked students to work with strategies other than halving and combine their halving strategy with new strategies. After assigning students to the task cards on the basis of their readiness levels, Sally monitored student readiness and understanding and had students trade in their cards for more difficult cards when necessary.

Sally was able to use a number of the problem situations for more than one level of the task. For example, “How can 3 people share 5 brownies?” was used as a more challenging problem for middle-level learners to continue working toward strategies that do not use repeated halving and was also used as a base-level question for the high-level learners to start from as they moved to the more difficult process of dividing brownies into fifths. To help students understand that they were all working on similar concepts and all had important knowledge to contribute during discussions, Sally regrouped them to share their solution paths. She also used the “overlap” problems in the whole-class closure activities.

## Figure 2

**Creating multiple versions of a task provides equitable access to understanding the fraction unit.**

**Solve these problems on Student Sheet 3:**

- How can 2 people share 3 brownies?
- How can 2 people share 5 brownies?
- How can 4 people share 3 brownies?
- How can 3 people share 4 brownies?

**Solve these problems on Student Sheet 3:**

- How can 4 people share 3 brownies?
- How can 3 people share 4 brownies?
- How can 3 people share 5 brownies?
- How can 6 people share 4 brownies?

**Solve these problems on Student Sheet 3:**

- How can 3 people share 5 brownies?
- How can 3 people share 2 brownies?
- How can 6 people share 4 brownies?
- How can 5 people share 4 brownies?

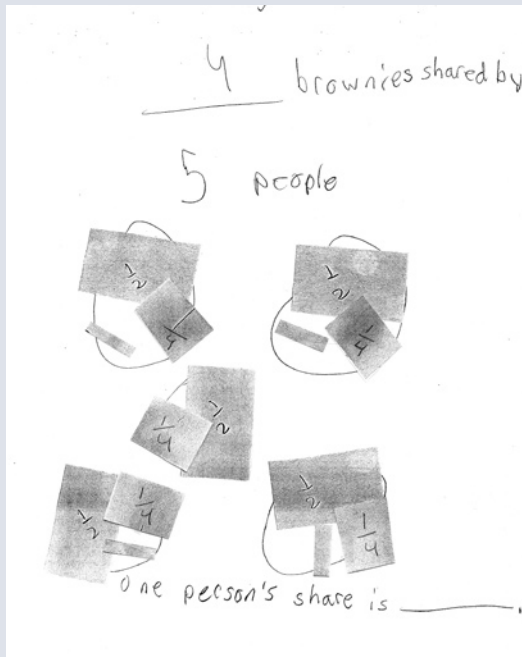
## Thinking about Equity

Sally’s practices supported equitable access to what she had identified as the important understandings for her fraction unit by asking all students to build on their current understandings and use strategies to solve authentic problems. All the situations on the task cards required students to work with the important mathematical idea that fractions are equal parts of a whole, and all had students work with shares that were both greater than and less than one. All the recordings asked students to make sense of the symbolic representations of fractions that were connected to pictorial representations of fractions.

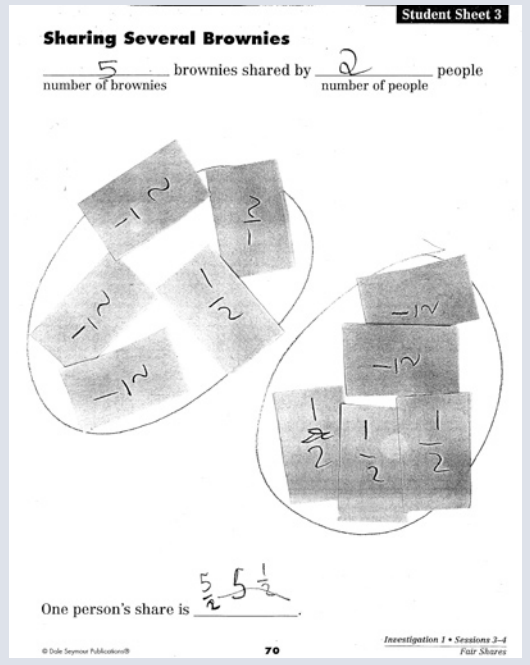
In this lesson, Sally found it necessary to provide the scaffold—paper brownies—to all students. The most difficult problem, the challenge of sharing four brownies among five people, could have been easy if the students had discovered the strategy of dividing each brownie into fifths. However, Sally’s third graders, like most typical third graders, used halving first and gave each person half of a brownie (Tierney and Berle-Carman 1998), using two and one-half brownies and leaving one and one-half brownie to still be

**Figure 3**

(a) A student's pictorial solution to "How can 5 people share 4 brownies?"



(b) A student's pictorial solution to "How can 2 people share 5 brownies?"



divided. This left an awkward sharing situation (see **fig. 3a**). Having paper brownies to cut, fold, and make mistakes with allowed the students to deepen their conceptual understanding of fractional parts, make connections to their prior knowledge, and deepen their understandings (see **fig. 3b**.)

During the closing discussion, Sally was able to help the class as a whole move toward deeper understandings of the key concepts being studied. All students were in a situation where they could share their strategies for making equal parts and focus on questions such as, "Do some strategies work better with certain sharing situations? Why?" or "Is there a strategy that works well all the time? Why? How is that strategy related to the original problem?" (Dividing each brownie into enough pieces for each sharer relates to the division meaning of fractions: 3 brownies for 2 people =  $3/2$  for each person.)

Asking students questions such as, "How do you know what to use as a numerator or denominator?" leads them to generalizations about the relationships between their pictorial and symbolic representations. Students come to realize that denominators represent the number of equal pieces the wholes were divided into or that when they add like fractions, the denominator stays the same and the numerators are added together.

This type of closing discussion helps students understand that a variety of strategies can be used to solve problems. It also helps them see the relationship of numerators and denominators to each other and to the original problem. All the students had experiences relevant to the discussion, a clear indication that the exercise supported equity.

## Tiering a Second-Grade Geometry Lesson

Applying the same process to a second-grade geometry lesson provides an example of a tiered lesson that requires a variety of scaffolds, rather than the same scaffold, to support learners at different levels. In this case, Kevin, a second-grade teacher, identified the important mathematical idea for students to learn: He planned to help second graders understand that there are relationships between the attributes of a three-dimensional figure and the attributes of its net. (A *net* is a two-dimensional pattern that can be folded to cover a three-dimensional figure [Findell et al. 2001, p. 27]). Kevin anticipated a continuum of readiness among the second graders' knowledge of the components of three-dimensional shapes (e.g., the shapes of the faces, the number of edges, etc.) and a continuum of readiness among

**Figure 4**

Kevin's students enjoyed creating "Rolling Nets."



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the children with respect to their ability to take apart, or decompose, three-dimensional shapes into their component parts.

Kevin found that "Rolling Net" from *Navigating through Geometry in Prekindergarten–Grade 2* (Findell et al. 2001) provided a base activity for working on the key idea that a relationship exists between the attributes of a three-dimensional object and the attributes of its net. In this activity, students were asked to create the nets for objects by rolling three-dimensional shapes on a paper and tracing each face of the shape when it rested on the paper (see fig. 4.) This was an engaging task for students because it allowed them to use blocks and to draw. Students also enjoyed working with partners: One child held the block while the other traced each face.

When thinking about the complexity of this task with respect to the readiness of his students, Kevin asked the same questions Sally did:

- Who will find this task too easy? Why?
- Who will find it too difficult? Why?

The students like Joe, who have strong visual-spatial skills, were the ones who would be most likely to find this task quite easy and would be able to predict what the net would look like before creating it. Students with weak visual skills or with few experiences with blocks would find this task more difficult.

Developing different versions of the task to accommodate these readiness differences, Kevin considered the continuum of the students' visual-spatial strengths. In this case, he used a variety of three-dimensional shapes at three centers to provide appropriate levels of challenge (see fig. 5). Students with few experiences with blocks or with weak visual-spatial skills worked with cubes and then moved to other rectangular prisms. At a second station, students with stronger visual skills traced a variety of rectangular prisms, moved to triangular or hexagonal prisms, and finally tried a pyramid. Those students with very strong visual-spatial skills were assigned to work at a third station that contained a variety of pyramids, cylinders, and cones.

## Scaffolding the Geometry Task

Kevin anticipated that a second group of students, those who lacked fine motor skills, would also have difficulty with this task. Although weak fine motor skills would not affect the students' ability to comprehend the important mathematical ideas

**Figure 5**

A variety of three-dimensional shapes at three separate centers provided levels of challenge for second-graders.



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or to predict the shape that resulted from the nets, they would decrease a student's chances of being successful with the task. Kevin provided supports, or scaffolds, for these students in two ways. First, these students were given the largest blocks available for the task. Some of these were available from the second-grade materials; others he borrowed from kindergarten and first-grade classrooms. Second, he provided graph paper as a scaffold. It allowed students to follow lines when they rolled their cubes and rectangular prisms and enabled more accuracy when the students were tracing.

Students with the cylinders and cones needed a scaffold as well. The rounded surfaces provided a unique challenge when students traced. Kevin drew a line on the side of the cylinder as a clue to mark the starting or stopping point for tracing. Also, Kevin anticipated a need for a guiding question (such as, "How do you suppose the line can help you as you are tracing?") and asked the question to support students as they were looking for strategies for recording the more complex nets.

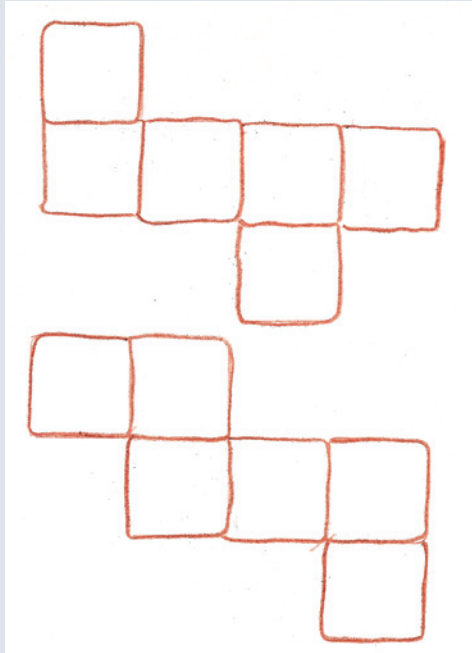
Finally, all students were provided with a visual model of "rolling" a cube to ensure that the shapes in the net were connected and to demonstrate that problem solving would be needed when nets were close to the edge of the paper. All students were also given stickers (to mark the surfaces traced) and the option to work with a partner (if holding the object while tracing it was too difficult).

## Equity Revisited

As with Sally's fraction activity, Kevin's lesson supported equitable access to important mathematics. All the students participating in this differentiated version of "Rolling Net" solved the problem of how to "roll" the three-dimensional objects in order to accurately record their nets. All the students worked with blocks and tracing during the problem-solving process. During the closing discussion, Kevin collected data from all the students' experiences to determine relationships between the attributes of each three-dimensional object and its nets. For example, he created a chart comparing the number of faces on the three-dimensional objects to the number of shapes in the nets. This helped students generalize that the number of faces was always equal to the number of shapes drawn in the net. Comparing nets made by different pairs of students for the same three-dimensional shapes demonstrated that more than one net could be drawn for each shape (see **fig. 6**) and led students to ask how

**Figure 6**

Two nets for the same cube.



many nets were possible for each object. This question was left to be explored in a mathematics center in future classes.

## Conclusion

By identifying the important mathematics to be taught and then creating solid, tiered tasks that required higher-level thinking, Sally and Kevin engaged all students in activities that provided opportunities to increase their level of understanding of key ideas within the mathematics curriculum. Scaffolding tasks allowed students to work independently at appropriately challenging levels, make sense of ideas, and develop a sense of self-confidence in their mathematics knowledge and skills.

Students bring a variety of background knowledge, understandings, and experiences to the mathematics classroom. Ignoring that fact and teaching them as though they are all the same would be inequitable. Strategies such as tiering and scaffolding allow teachers to design a variety of paths to understanding that, in turn, create a more equitable mathematics classroom.

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*All photos in this article are of children in a combined summer school class of second and third graders.*

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